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Inhibition in Speed and Concentration Tests: The Poisson Inhibition Model

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A new model is presented to account for the reaction time fluctuations in concentration tests. The model is a natural generalization of an earlier model, the so-called Poisson–Erlang model, published by Pieters & van der Ven (1982). First, a description is given of the type of tasks for which the model has been developed. Next, the new model, called the Poisson inhibition model, is described. Each reaction time is considered as series of alternating processing times and distraction times. During processing, the transition rate from work to distraction is assumed to be constant. Therefore, the number of distractions has a Poisson distribution. During distraction, the transition rate from rest to work is inversely related to the level of inhibition. The model is a limiting case of a model in which inhibition is assumed to oscillate between a lower and an upper limit. The present model is described in such a way that computer simulations of the reaction times can be made. Furthermore, the moments of the reaction times are derived. At the end of the paper it is shown that a description of the actual time series in terms of the underlying inhibition process is in complete agreement with Spearman's theory about the universal factors, which are the general factor and the factors oscillation and perseveration.

INTRODUCTION

Pieters & van der Ven (1982), van der Ven & Smit (1982), van Breukelen, Jansen, Roskam, van der Ven & Smit (1987), and van der Ven, Smit & Jansen (1989) have developed several models to account for the reaction time fluctuation in concentration tests, which consist of overlearned continuous response tasks, also referred to as overlearned prolonged work tasks. In this article a new model is presented, which is a natural generalization of an earlier model, the so-called Poisson–Erlang (PE) model, which was published by Pieters & van der Ven in 1982. First, this paper describes the type of tasks for which the models have been developed. Next, a short overview is given of the models which have been developed in the past. Following that, the new model, called the Poisson inhibition model, is described. Finally, it is shown that the interpretation of the main descriptive properties of the time series in terms of the inhibition model is in complete agreement with the universal factors postulated by Spearman, which are **g** and oscillation and perseveration.

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EXPERIMENTAL PARADIGM

The model to be presented here applies to the speed component both in so-called *speed* tests and in *concentration* tests. The development of speed tests mainly originated from the Anglo-American tradition of intelligence measurement, whereas concentration tests come from the European tradition. In contemporary intelligence measurement, speed tests are time-limit tests in which all items can be solved if the subjects are given unlimited time. Although the items are relatively easy, they still may vary in difficulty. This means that some items require more time than others. Typical examples of present day speed tests are Name Comparison, Tool Matching, Form Matching, and Computation from the General Aptitude Test Battery, and Reasoning and Number from the Primary Mental Abilities Tests. It is the number of items correctly solved, given the time limit, that provides the index of speed. This index, however, is dependent not only on speed of work, but also on the difficulty of the items. In order to study speed of work as such, and especially its random variability, the items should be of the same difficulty and an attempt should be made to time individual items. This approach is used in conventional speed tests. The conventional speed tests required subjects to engage in repetitive activities, such as letter cancellation, detecting differences in simple shapes, adding three digits, and so on. In the majority of studies however, no attempt was made to time individual items. At the same time, exactly the same type of tests were used in Europe. However, here, the duration of individual items or groupings of items were measured and employed in the assessment of subject performance. These tests, which are actually speed tests in the conventional meaning of the word, were refered to as concentration tests. The difference is not in test content or test instruction (Work as quickly and as accurately as possible!), but in performance registration. Instead of one gross measure such as number of correct items or total time needed to complete the test, individual item scores such as the time needed to complete each

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separate item or groupings of items were used. The characteristics of these two distinct trends in speed measurement can be summarized in terms of differences between timing procedures used and not in terms of test content. The model to be discussed here requires the timing of individual items or groupings of items, which is the case in concentration tests. Therefore, these tests will be given more attention.

Concentration tests were already used in the very beginning of this century. Binet (1900), for example, the author of the well-known intelligence scale, reports an extensive study on the measurement of concentration. He refers to it as "la force d'attention volontaire." He made use, among others, of a so-called letter cancellation test originally proposed by Bourdon (1895). This test consisted of crossing five letters, such as the vowels of the alphabet, in a meaningful text for 10 min. For each 1-min. period the number of crossed letters and the number of errors was recorded. Binet was well aware of the importance of the fluctuation in speed and error suggesting the mean deviation as a measure of performance. However, in reporting the final results he reported only general level scores such as the mean number of crossings and the mean number of errors during the first and second 5-min. period. Moreover, the test was still subject to learning. However, concentration tests should be overlearned in advance, because the purpose of the test is to measure ability to concentrate and not learning ability (as is the case in reminiscence tasks, see below). Later, Abelson (1911) reported a study on the measurement of mental ability of "backward" children. As in the study of Binet, only total scores i.e., the total time to complete the test, were reported.

Subsequent researchers became increasingly aware that concentration test tasks should be relatively easy. Learning effects should be avoided and the relevant information should be found in the short term oscillation of the measure of performance. Godefroy (1915) was probably the first to stress the importance of the fluctuation in the response times. He used the mean deviation of the response time as an indication of concentration. Spearman, several decades later, considered even oscillation to be a separate universal factor; in addition to the general factor, g, and perseveration (Spearman, 1927, p. 327). A typical manifestation of this factor "...is supplied by the fluctuations which always occur in any person's continuous output of mental work, even when this is so devised as to remain of approximately constant difficulty." (Spearman, 1927, p. 320). According to Spearman "...almost any kind of continuous work can be arranged so as to manifest the same phenomenon. In all cases alike, the output will throughout exhibit fluctuations that cannot be attributed to the nature of the work, but only to the worker himself." (Spearman, 1927, p. 321).

At present the typical concentration test consists of a simple mental task such as addition of one digit numbers, cancellation of letters, crossing out sets of dots, etc. The task has to be performed for a relatively long period of time varying from 10 to 30 min. Performance is measured by a time series that consists of either a series of *response times*, in which each response time is the result of a fixed number of responses, or a series of *response counts*, in which each count is obtained in a fixed amount of time. A well-known example of the former is the Bourdon–Vos test (Vos, 1988), which is a children's version of the Bourdon–Wiersma test (see Huiskam & de Mare, 1947, and Kamphuis, 1962) used in The Netherlands. A well-known example of the latter is the Pauli test (see Arnold, 1975) used in Germany, which is a single-digit addition task. The time series consists of the number of additions per minute during a 30-min. period. This article is limited to models for response times. In principle it is possible to change these models into models for response counts.

In concentration tests the subject responds in a self-paced continuous manner. The person controls his own speed: the subject's response to each part releases the next one in the sequence. He/she is not allowed to take rest pauses between parts. This may result in a certain dependency of the item response times. The mental state of a subject at the start of a recording may be dependent on the mental state of the subject at the end of the previous recording. Consequently the consecutive item response times should be studied as a time series.

TASKS OUTSIDE THE THEORY

Prolonged work tasks also occur in experiments on *reminiscence*. The study of reminiscence has also a long history, which is briefly described in Eysenck & Frith (1977, Chap. 1). "Reminiscence is a technical term, coined by Ballard in 1913, denoting improvement in the performance of a partially learned act that occurs while the subject is resting, that is, not performing the act in question." (Eysenck & Frith, 1977, p.3). The reality of the phenomenon was first experimentally demonstrated by Oehrn (1896). In experiments on reminiscence the same task is always administered twice or more. Learning curves are obtained which usually include a pre-rest period of massed practice, a rest period, and a post-rest period. The tasks which are used are highly sensitive to learning. One is mainly interested in long-term trend effects, disregarding the short-term fluctuations of the individual response times. In contrast with reminiscence tasks, concentration tests typically consist of tasks which are already familiar to the subject before administration. Usually some practice trials are given before the actual test is administered in order to eliminate any remaining learning effects. In concentration tasks one is mainly interested in the short-term fluctuations of the response times. In reminiscence tasks the interest is primarily in the long-term trend. Although the tasks used in experiments on reminiscence are prolonged work tasks

similar to the tasks used in concentration tests, they do not belong to the domain of tasks that form the object of this study. The models which will be discussed in this paper apply only to concentration tests which consist of *overlearned*, prolonged work tasks, that is, tasks in which learning effects can be disregarded. However, it is quite possible to enrich these models by assuming that some of the parameters may change, dependent on practice.

Additionally, *vigilance* tasks are not the object of research in this study. In vigilance tasks the subject is required to keep watch for *inconspicuous* signals (either visual or auditory) over long periods of time (one hour or more). Systematic scientific investigation of vigilance was initiated by Mackworth (1950), who simulated the task of maintaining radar watch for submarines by using a clock pointer which moved on a series of steps. The subjects watched the pointer and reported the relatively *infrequent* occasions on which the pointer gave a double jump. "The most important finding is the so-called vigilance decrement: the probability of signal detection tends to decrease over time." (Eysenck, 1982, p. 80). Unlike vigilance tasks, concentration tests consist of stimuli (or items), which are presented over short periods of time (10 to 20 min.), each requiring a response. Responses occur frequently, instead of infrequently as in vigilance tasks. In vigilance research one is mainly interested in studying the effect of fatigue or boredom. In concentration tests, the task should be completed before fatigue or boredom may play a role. However, the models which will be discussed in this study can be adapted by allowing the relevant parameters to change, dependent on fatigue or boredom.

INHIBITION AS AN EXPLANATORY CONCEPT

Pieters and Van der Ven (1982) introduced the explanatory concept of distraction to account for the response time fluctuations occurring in concentration tests. They assumed that the manifest response time should be considered to be composed of a relatively constant (over items or groupings of items) real total work time, interrupted by a series of random distraction times. This led to the formulation of the Poisson–Erlang model, which is based on the following three assumptions: (1) each separate distraction time has an exponential distribution, (2) the number of distractions has a Poisson distribution, and (3) the real total working time is constant over responses. The notion of intermediate periods of distraction has already been suggested by many authors; see Peak & Boring (1926), Bills (1931, 1935, 1964), and Berger (1982).

The Poisson–Erlang model is able to account for the short-term variation in the response times, but not for any long-term trend effect, although in actual time series, this effect usually does occur. In many cases at the beginning of the test, there is an increase in the reaction times, which in the long run starts fluctuating at some stationary level. Pieters & van der Ven (1982) and Pieters (1985) always used some correction procedure to remove the long-term trend from the data, before actually testing the model. However, since this phenomenon occurs very frequently, it is much more plausible to assume that the emergence of a long-term trend is a part of the distraction process itself.

In the past few years two alternative models have been developed in order to account for the long-term trend and the possible interdependency of the response times (van der Ven & Smit, 1982, and van der Ven *et al.* 1989). Both models are based on the assumption that distractions are periods of recovery from accumulated inhibition. The general assumption that during work inhibition increases and during distraction inhibition decreases is made. The 1989 model is mathematically simple in comparison to the 1982 model. In the 1982 model, individual working and distraction times both are dependent on the underlying process of inhibition increase and decrease. In the 1989 model only the individual working times are assumed to be dependent on inhibition. A reparametrisation of the 1989 model is given by Roskam (see van Breukelen *et al.* 1987). In the model to be presented in this article only the individual distraction times are assumed to be dependent on inhibition.

The Poisson inhibition model, as well as previously developed inhibition models, is developed to account for the sequence of RTs which can be observed when a single test is administered to a single subject. The response (or reaction) time T, which the subject needs for a particular response in the reaction time sequence, is modelled as the sum of individual working and distraction times. The reaction time T is *observed*, whereas the individual working and distraction times are *latent*.

THE POISSON INHIBITION MODEL

The model is designed to explain the statistical properties of a series of reaction times, $T_1, T_2, \ldots$, representing the amounts of time a person uses on each one of a sequence of tasks executed consecutively. As previously mentioned, it is assumed that each task requires for its completion the same amount of processing time, A. The actual time T spent on a task (the reaction time), exceeds A because of distractions interrupting work on the task. So, $T = A + D$, where $D = d_1 + d_2 + \cdots + d_N$ is the sum of the distraction intervals during this particular task. The durations d_i of these distractions are random variables and so is their number N. (No special notation such as boldface or capital letters to denote random variables will be used. It will be clear from the context which variables are random variables.)

In an earlier model, the PE model, N has a Poisson distribution and the d_i are exponentially distributed, so D has an Erlang distribution.

In the PE model the successive reaction times constitute a sequence of independent random variables with the same distribution. Trend phenomena, often observed in actual reaction time series, cannot be explained by this model. In order to get a more adaptable model the above situation is described as follows.

The person is alternately in state 1, processing or work, and in statue 0, distraction or rest. Let $X(t)$ denote the state the person is in at time t. Assuming $X(t)$ to be a two state continuous time Markov process (see Hoel, Port, & Stone, 1972), with constant transition rates λ_1 and λ_0, leads to the PE model (transition rates: When in state i at time t, the probability of jumping to the other state in the infinitesimal time interval $(t, t+dt)$ is given by $\lambda_i\, dt$). The intended adaptation of the model is obtained by allowing the transition rates to vary according to the level of inhibition. Inhibition, like fatigue, increases during processing and decreases during distractions. This idea has already been suggested by Spearman when he relates oscillation in performance to what he assumes to be an alternating process of energy consumption (read inhibition increase) and energy recuperation (read inhibition decrease) (see Spearman, 1927, p. 327). The transition rate λ_0 and λ_1 are assumed to change with the level of inhibition in such a way that when inhibition is high, distractions will be long relative to the length of work intervals. This causes inhibition to decrease. Likewise when inhibition is low, distractions will be relatively short and as a result inhibition will rise. This makes it plausible that inhibition will tend to behave like a stationary process, fluctuating around a central region and tending to return to this region whenever it finds itself outside of it. The possibility of trend phenomena in the series of reaction times is thus easily perceived. For example, if the initial inhibition happens to be low, one will have short distractions (and hence short reaction times) in the beginning. As a consequence, the inhibition gradually increases and this causes distractions (and hence also reaction times) to become longer. So, one gets an upward trend in the reaction time series. The opposite phenomenon, a downward trend in the reaction time series, is to be expected when the initial inhibition is high (relative to its stationary mean value).

The exposition of the mathematical model consists of the following subsections:

1. Assumptions of the general inhibition model, the Poisson inhibition model, and the beta inhibition model.
2. Simulation of the model.
3. The inhibition process $Y(t)$, stationary distribution.
4. Moments of $Y(t)$.
5. Moments of reaction times (stationary case).
6. Trend and autocorrelation.

1. *Assumption of the General Inhibition Model, the Poisson Inhibition Model, and the Beta Inhibition Model*

The assumptions of the general inhibition (GI) model are as follows:

GI1. The person is alternately in state 1, processing, or in state 0, distraction: $X(t)$ denotes state at time t. Each task requires the same amount of processing time A. So the successive tasks are completed at the moment $T_1, T_1+T_2, T_1+T_2+T_3, \ldots$, when the accumulated processing time reaches $A, 2A, 3A, \ldots$ ($T_1, T_2, \ldots$ constitute the series of reaction times).

GI2. Inhibition denoted by $Y(t)$ increases linearly with rate a_1 during work: $Y'(t)=a_1$ when $X(t)=1$ and decreases linearly with rate a_0 during distractions: $Y'(t)=-a_0$ when $X(t)=0$.

GI3. The transition rates λ_1, from state 1 to state 0, and λ_0, from state 0 to state 1, depend on inhibition: $\lambda_1=l_1(Y)$, $\lambda_0=l_0(Y)$, where l_1 is a non-decreasing function and l_0 is a non-increasing function.

Specification of the functions l_1 and l_0 leads to various "special inhibition models." One of these models, the Poisson inhibition model, will be elaborated in the sequel. In this model l_1 and l_0 are as follows:

$$l_1(y)=c_1 \qquad \text{(positive constant)}. \tag{1}$$

In the Poisson inhibition (PI) model the transition rate from state 1 to state 0 is constant. Since a task requires for its completion (by assumption GI1) an amount of working time A, and during this time interruptions occur with rate c_1, it follows that the number of distractions is Poisson distributed with mean c_1A. This was the reason for the "Poisson" in the name of the model:

$$l_0(y)=c_0/y \qquad \text{for} \quad y>0 \quad (c_0 \text{ a positive constant}). \tag{2}$$

The transition rate from state 0 to state 1 is given by $\lambda_0(t)=l_0(Y(t))=c_0/Y(t)$. Note that as $Y(t)$ goes to zero (during a distraction), the transition rate $\lambda_0(t)$ goes to infinity and this forces a transition to state 1 before the inhibition could reach zero.

In order to ensure that $Y(t)$ is always positive the additional assumption that when a person is not performing a task, his/her mind is alternating between states 1 and 0, with transition rates given by the same functions $\lambda_i(t)=l_i(Y(t))$, $i=0, 1$ is also made. However, the rate of increase of $Y(t)$ when $X(t)=1$ during a leisure period is less (say a_2) than a_1, the rate of increase when $X(t)=1$ during performance of the task. As a result the mean level of inhibition during leisure periods is lower than it will be during tasks, as will be demonstrated in subsection 3.

The special choice of the functions l_0 and l_1 in the PI model is motivated mainly by mathematical convenience. This choice makes it possible to derive exact expressions for the moments and the correlation function of the $Y(t)$ process and the reaction time sequence. Other choices may produce models with roughly the same behavior. One such model has $l_1(y) = c_1/(1 - y/M) = c_1 M/(M - y)$ and $l_0(y) = c_0/y$. This model has $Y(t)$ fluctuating in the interval between 0 and M. The stationary distribution for $Y(t)/M$ in this case is a beta distribution (reason to call it the beta inhibition (BI) model). The reader may check this assertion for her- or himself following the steps set out in subsection 3 where the stationary distribution of $Y(t)$ for the PI model is shown to be a gamma distribution. So the PI model might as well be called the gamma inhibition model. For $M \to \infty$ the beta inhibition model converges to the PI model.

In order to reduce the number of parameters of the PI model (five parameters: A, a_0, a_1, c_0, and c_1), one might fix $a_0 = 1$. That is, the assumption is made that inhibition decreases with rate 1 during distractions. Since inhibition is not a directly observable (or measurable) entity this assumption is quite reasonable. It amounts to measuring inhibition in time units of distraction necessary to reduce it.

2. *Simulation*

2a. In order to understand how the model works, it is helpful to use the constructive approach and explain how to do computer simulations of the process involved. To this end one needs to know the distribution of the lengths of work intervals and distraction intervals. Since these intervals are terminated by sudden transitions to another state, first the general problem of finding the *distribution of the waiting time T to the first transition when the transition rate $\lambda(t)$ changes with time* is considered. Let F be the distribution function of T. The function $G(t) = 1 - F(t) = P(T > t)$ satisfies

$$\begin{aligned} G(t+h) &= P(T > t+h) \\ &= P(T > t)\, P\,(\text{no transition in } (t, t+h)) \\ &= G(t)(1 - \lambda(t)\, h) \qquad \text{for } h \text{ (infinitesimally) small.} \end{aligned}$$

This yields

$$\frac{G(t+h) - G(t)}{hG(t)} = -\lambda(t).$$

with h tending to zero one gets

$$\frac{G'(t)}{G(t)} = \frac{d}{dt}(\ln G(t)) = -\lambda(t).$$

So, by integrating $-\lambda(t)$ one may obtain $\ln G(t)$. Considering the initial value $G(0) = 1$, $\ln G(0) = 0$, one gets $\ln G(t) = -\int_0^t \lambda(s)\, ds$ and hence

$$G(t) = P(T > t) = \exp\left[-\int_0^t \lambda(s)\, ds\right]. \tag{3}$$

To obtain simulated values of the random variable T with distribution function F, a standard method is applied. Solve the equation $G(t) = U$ for t, where $G = 1 - F$. This yields $t = G^{-1}(U)$, where G^{-1} is the inverse function of G (which is assumed to exist). Now, if U is a uniform (0, 1) random variable (the usual random number), then the random variable $G^{-1}(U)$ has the same distribution as T. This is obvious from the following:

$$P(G^{-1}(U) > t) = P(U < G(t)) = G(t).$$

Note the reversal of the inequality sign caused by the fact that G is a decreasing function; remember $G(t) = P(T > t)$. This method is sometimes called the inverse distribution function method, since solving $G(t) = U$ for t is equivalent with solving $1 - F(t) = U$ or $F(t) = 1 - U = V$, yielding $F^{-1}(V)$, where $V = 1 - U$ is also a (0, 1) random variable.

2b. *Work intervals. For a work interval w starting at t_0 with inhibition level $y = Y(t_0)$ the transition rate at time* $t_0 + t$ is given by $\lambda(t) = l_1(y + a_1 t)$. Since by (3) one has to integrate this transition rate, it is convenient to introduce the integrated forms (or primitive functions) L_1 (and L_0) of l_1 (and l_0), which satisfy $L_1'(y) = l_1(y)$ (and $L_0'(y) = l_0(y)$).

Integrating $\lambda(t) = l_1(y + a_1 t)$ from 0 to t gives $(1/a_1)(L_1(y + a_1 t) - L_1(y))$ and hence by (3) one gets

$$P_y(w > t) = \exp[-(1/a_1)(L_1(y + a_1 t) - L_1(y))], \tag{4}$$

where the subscript y indicates that this probability is conditional on the inhibition level y at the start of this work interval. To obtain simulated values for w one has to invert G defined by $G(t) = P_y(w > t)$ to obtain $w = G^{-1}(U)$ as an expression to produce simulated work intervals.

EXAMPLE a. In the PI model $l_1(y) = c_1$, so $L_1(y) = c_1 y$ and $G(t) = P_y(w > t) = \exp(-c_1 t)$. So work intervals may be simulated by $w = G^{-1}(U) = -(1/c_1) \ln U$.

EXAMPLE b. In the BI model one has $l_1(y) = c_1 M/(M - y)$, $L_1(y) = -c_1 M \ln(M - y)$. Substitution in (4) gives $P_y(w > t) = ((M - y - a_1 t)/(M - y))^{c_1 M/a_1} = G(t)$. Simulation with $w = G^{-1}(U)$ yields $w = a_1^{-1}(M - y)(1 - U^{a_1/c_1 M})$.

Note that the increase in inhibition $a_1 w$ during this work interval is always less than $M - y$, which ensures that the inhibition in this model always remains below the upper bound M.

2c. *Distraction intervals.* In a distraction interval starting at time t_0 at inhibition level $y = Y(t_0)$, the transition rate is given by

$$\lambda(t) = l_0(y - a_0 t).$$

Integrating from 0 to t gives $-(1/a_0)(L_1(y - a_0 t) - L_1(y))$, so by (4) one gets

$$P_y(d > t) = \exp[(1/a_0)(L_0(y - a_0 t) - L_0(y))]. \qquad (5)$$

EXAMPLE. In the PI model (and the BI model) $l_0(y) = c_0 y^{-1}$, $L_0(y) = c_0 \ln y$. Substituting these functions in (5) gives

$$\begin{aligned} P_y(d > t) &= \exp[c_0/a_0(\ln(y - a_0 t) - \ln(y))) \\ &= [(y - a_0 t)/y]^{c_0/a_0}. \end{aligned}$$

By inversion of G (where $G(t) = P_y(d > t)$) one obtains the expression for the simulation of d $(d = G^{-1}(U))$: $d = (1/a_0)(1 - U^{a_0/c_0})\, y$.

Note that the amount by which inhibition is diminished during distraction $a_0 d = (1/a_0)(1 - U^{a_0/c_0})\, y$ is always less than y, the inhibition at the beginning of the distraction. This prevents inhibition from becoming negative. The mean value of the decrease in inhibition $a_0 d$ is calculated by integrating $a_0 d = (1/a_0)(1 - U^{a_0/c_0})\, y$ over all values of U between 0 and 1. This results in

$$E_y(a_0 d) = (c_0 a_0^{-1} + 1)^{-1}\, y. \qquad (6)$$

2d. *Simulation of reaction times (for the PI model).* Let $y_0 = Y(0)$ be given, the initial inhibition. The first work interval w_1 is generated by $w_1 = -(1/c_1) \ln U$. The subsequent distraction interval d_1 is then generated by $d_1 = (1/a_0)(1 - U^{a_0/c_0})\, y$, with $y = y_0 + a_1 w_1$. The second work interval w_2 is generated like w_1. The second distraction d_2 is generated like d_1 with $y = y_0 + a_1 w_1 - a_0 d_1 + a_1 w_2$ and so on. To obtain a series of reaction times, one keeps track of the accumulated work time. The first reaction time is given by $T_1 = A + D_1$, where D_1 is the sum of the distractions occurring before the accumulated processing time reaches A. Likewise $T_k = A + a_0 D_k$ with D_k the sum of the distractions occurring between the moments when the accumulated work time reaches $(k-1)\, A$ and kA.

3. *The Inhibition Process $Y(t)$*

Since the completion of tasks is determined by the accumulated working time, it is convenient to adopt as *time parameter* for the process $Y(t)$ not real time but *accumulated working time*. With this convention, the evolution of the process $Y(t)$ is as follows. Let $Y(0)$ be the initial value of the inhibition (random or fixed). During the first work interval (duration w_1), $Y(t)$ increases linearly with rate a_1. So, for $0 \leqslant t \leqslant w_1$, one has $Y(t) = Y(0) + a_1 t$. At $t = w_1$ there is a discontinuous downward jump of magnitude ad_1 caused by the first distraction (duration d_1). During the second interval, $Y(t)$ rises again: for $w_1 < t < w_1 + w_2$ one has $Y(t) = Y(0) + a_1 w_1 - a_0 d_0 + a_1(t - w_1) = Y(0) + a_1 t - a_0 d_0$. During the third work interval $(w_1 + w_2 < t < w_1 + w_2 + w_3)$, $Y(t) = Y(0) + a_1 t - a_0(d_1 + d_2)$, and so on. At completion of the first task, $t = A$ and

$$Y(A) = Y(0) + a_1 A - a_0 D_1, \qquad (7)$$

where D_1 is the total length of the distraction occurring while working the first task. So, $D_1 = a_1 a_0^{-1} A - a_0^{-1}(Y(A) - Y(0))$, and since $T_1 = A + D_1$,

$$T_1 = A + a_1 a_0^{-1} A - a_0^{-1}(Y(A) - Y(0)). \qquad (8)$$

In the same way the nth reaction time T_n equals

$$T_n = A + a_1 a_0^{-1} A - a_0^{-1}(Y(nA) - Y((n-1)\, A)). \qquad (9)$$

So, one sees that the sequence of reaction times $T_1, T_2, \ldots$, is determined by the increments of the process $Y(t)$ over the (working) time intervals 0 to A, A to $2A$, ...

As one wants to study the behavior of the raction time series, one needs to investigate the distribution of the inhibition process.

Let $F(y, t) = P(Y(t) \leqslant y)$ and $f(y, t) = (d/dy)\, F(y, t)$ denote the distribution and density function of $Y(t)$. One has the following equation for small h:

$$\begin{aligned} &F(y + a_1 h, t + h) \\ &\quad = F(y, t) + \int_y^{\infty} f(u, t)\, l_1(u)\, h P_u(a_0 d > u - y)\, du. \qquad (10) \end{aligned}$$

By way of explanation, the event "$Y(t+h) \leqslant y + a_1 h$" will certainly occur if already "$Y(t) \leqslant y$" occurs. Moreover, it may occur when $Y(t) = u$ with u somewhere between y and ∞, provided a downward jump occurs (probability $l_1(u)\, h$); moreover, this downward jump in the inhibition should be at least of magnitude $u - y$ to ensure that $Y(t+h)$ is less than $y + a_1 h$. The probability

that the jump is large enough is given by $P_u(a_0 d > u - y)$. Making the proper substitutions in formula (5) (subsection 2c) yields

$$P_u(a_0 d > u - y) = \exp(a_0^{-1}(L_0(y) - L_0(u))).$$

So (10) becomes

$$\begin{aligned} &F(y + a_1 h, t + h) - F(y, t) \\ &\quad = h \exp(a_0^{-1} L_0(y)) \int_y^\infty f(u, t)\, l_1(u) \\ &\qquad \times \exp(-a_0^{-1} L_0(u))\, du. \end{aligned} \tag{11}$$

Divide by h and let h tend to 0:

$$\begin{aligned} &a_1 F_y(y, t) + F_t(y, t) \\ &\quad = \exp(a_0^{-1} L_0(y)) \int_y^\infty f(u, t)\, l_1(u) \\ &\qquad \times \exp(-a_0^{-1} L_0(u))\, du. \end{aligned}$$

The subscripts y and t in F_y and F_t stand for partial differentiation with respect to y and t, respectively. Note that $F_y(y, t) = f(y, t)$. One more differentiation with respect to y gives

$$\begin{aligned} &a_1 F_{yy}(y, t) + F_{ty}(y, t) \\ &\quad = a_0^{-1} l_0(y)(a_1 F_y(y, t) + F_t(y, t)) - F_y(y, t)\, l_1(y). \end{aligned} \tag{12}$$

This partial differential equation might have (among other solutions) a time-independent solution (for which the derivatives with respect to t vanish). If such a solution exists (denoted by $G(y)$, with density $g(y)$), it should satisfy Eq. (12) with $F(y, t) = G(y)$, $F_y(y, t) = g(y)$, and $F_{yy}(y, t) = g'(y)$ while time derivatives are zero:

$$\begin{aligned} a_1 g'(y) &= a_0^{-1} l_0(y)\, a_1 g(y) - l_1(y))\, g(y), \\ g'(y)/g(y) &= a_0^{-1} l_0(y) - a_1^{-1} l_1(y), \end{aligned} \tag{13}$$

or

$$\begin{aligned} \ln g(y) &= a_0^{-1} L_0(y) - a_1^{-1} L_1(y) + C, \\ g(y) &= K \exp(a_0^{-1} L_0(y) - a_1^{-1} L_1(y)). \end{aligned} \tag{14}$$

Here K is normalization constant making g a probability density function. Whether g has a finite integral depends on the form of L_0 and L_1. In the PI and BI models the integral is finite.

In the case of the PI model with $L_0(y) = c_0 \ln y$, $L_1(y) = c_1 y$, one gets

$$g(y) = K y^{c_0 a_0^{-1}} \exp(-c_1 a_1^{-1} y). \tag{15}$$

This time-independent solution represents the so-called stationary distribution of the process $Y(t)$. If $Y(t)$ has this distribution at $t = t_0$, then $Y(t)$ has the same distribution for all $t \geqslant t_0$.

For the PI process (15) shows that (in the stationary case) $c_1 a_1^{-1} Y(t)$ has a gamma distribution with parameter $p = c_0 a_0^{-1} + 1$. So, $E(c_1 a_1^{-1} Y(t)) = p$ and $\mathrm{Var}(c_1 a_1^{-1} Y(t)) = p$. This implies that the stationary mean and variance of the process $Y(t)$ are given by

$$\mu = E(Y(t)) = c_1^{-1} a_1 (c_0 a_0^{-1} + 1)$$

and

$$\sigma^2 = \mathrm{Var}(Y(t)) = c_1^{-2} a_1^2 (c_0 a_0^{-1} + 1). \tag{16}$$

Note that μ is proportional to a_1, the rate of increase of inhibition in state 1. Presumably, this rate of increase is smaller during leisure periods (before and between the performance of tasks). So, before the person starts working on the task, the inhibition will fluctuate around a lower mean. When doing simulations one might draw the initial inhibition $Y(0)$ from a distribution with mean value less than μ.

4. *Moments and Covariance of the Inhibition Process*

Let $g(t) = E(Z(Y(t) - \mu))$, where Z is a function of $Y(0)$ and $\mu = a_1 c_1^{-1}(c_0 a_0^{-1} + 1)$, the stationary mean of $Y(t)$. It will be shown that

$$g(t) = g(0)\, e^{-bt} \qquad \text{with} \quad b = c_1 (c_0 a_0^{-1} + 1)^{-1} = a_1 \mu^{-1}. \tag{17}$$

To this end it is sufficient to show that g satisfies $g'(t) = -b g(t)$. Look at the difference $g(t + h) - g(t)$ for small h,

$$\begin{aligned} g(t + h) - g(t) &= E(Z(Y(t + h) - Y(t))) \\ &= E(Z E_t(Y(t + h) - Y(t))), \end{aligned} \tag{18}$$

where E_t stands for expectation given $Y(t)$. For small h one has $Y(t + h) - Y(t) = a_1 h - I a_0 d$, where the random variable I indicates whether or not a downward jump occurs in $(t, t + h)$. So $P(I = 1) = c_1 h$. If such a jump occurs, its

mean magnitude is given by (6) $E_t(a_0 d) = Y(t)(c_0 a_0^{-1}+1)^{-1} = Y(t)\, bc_1^{-1}$. So, one has $E_t(Y(t+h) - Y(t)) = a_1 h - c_1 h Y(t)\, bc_1^{-1} = h(a_1 - bY(t)) = -bh(Y(t)-\mu)$. This is substituted in (18); dividing by h, with $h \to 0$, results in

$$g'(t) = -bE(Z(Y(t)-\mu)) = -bg(t).$$

Here are two interesting applications of (17). The first one is with $Z=1$:

$$E(Y(t)-\mu) = E(Y(0)-\mu)\, e^{-bt}$$

or

$$E(Y(t)) = \mu + (E(Y(0)) - \mu)\, e^{-bt}. \tag{19}$$

This shows that $E(Y(t))$ tends to the stationary mean μ. The second application goes with $Z = Y(0) - m_0$ with $m_0 = E(Y(0))$. As a result one gets the covariance

$$\begin{aligned}\mathrm{Cov}(Y(0), Y(t)) &= E((Y(0)-m_0)(Y(t)-\mu)) \\ &= E((Y(0)-m_0)(Y(0)-\mu))\, e^{-bt} \\ &= \mathrm{Var}(Y(0))\, e^{-bt}.\end{aligned} \tag{20}$$

Of course, interdependence between $Y(0)$ and $Y(t)$ is the same with $Y(s)$ and $Y(s+t)$. So,

$$\mathrm{Cov}(Y(s), Y(s+t)) = \mathrm{Var}(Y(s))\, e^{-bt}. \tag{21}$$

With a little more effort one also may derive a rather complicated formula for $\mathrm{Var}(Y(t))$ from which it follows that it tends to the stationary variance as t increases.

5. *Moments of the Reaction Times (Stationary Case)*

In this subsection the assumption that the process $Y(t)$ is fluctuating stationary (as it might well do after running some time) is made. So, for all t one has $E(Y(t)) = \mu$, $\mathrm{Var}(Y(t)) = \sigma^2$, and $\mathrm{Cov}(Y(s), Y(s+t)) = \sigma^2 e^{-bt}$. From the relation

$$T_n = (1 + a_1 a_0^{-1})\, A - a_0^{-1}(Y(nA) - Y((n-1)\, A)) \tag{22}$$

one easily obtains mean, variance, and covariance of the reaction times:

$$E(T_n) = A(1 + a_1 a_0^{-1}) \tag{23}$$

$$\mathrm{Var}(T_n) = 2a_0^{-2}\sigma^2(1-r) \quad \text{with} \quad r = e^{-bA} \tag{24}$$

$$\mathrm{Cov}(T_n, T_{n+k}) = -a_0^{-2}\sigma^2 r^{k-1}(1-r)^2. \tag{25}$$

In particular, for $k=1$,

$$\mathrm{Cov}(T_n, T_{n+1}) = -a_0^{-2}\sigma^2(1-r)^2. \tag{26}$$

Dividing by the variance one gets the correlation between successive reaction times T_n and T_{n+1} (the autocorrelation lag 1 of the series):

$$\rho_1 = -\tfrac{1}{2}(1-r) = -\tfrac{1}{2}(1-e^{-bA})). \tag{27}$$

So, the PI model predicts negative correlation between successive reaction times.

6. *Trend Phenomena in the Reaction Time Series*

If $E[Y(0)] = m_0$ is not equal to the stationary mean μ then by (19), $E(Y(nA)) = \mu + (m_0 - \mu)\, r^n$ with $r = e^{-bA}$. So, by (9) this causes an exponential trend in the reaction time series:

$$E(T_n) = (1 + a_1 a_0^{-1})\, A + a_0^{-1}(m_0 - \mu)\, r^{n-1}(1-r). \tag{28}$$

This trend (in the expected values) is upward or downward dependent on whether $m_0 < \mu$ or $m_0 > \mu$. However, since $Y(0)$ is a random variable, the actual trend observable in a particular reaction time series depends on the particular value y taken by $Y(0)$. In fact, denoting conditional expectation given $Y(0) = y$ by E_y, one has

$$E_y(T_n) = (1 + a_1 a_0^{-1})\, A + a_0^{-1}(y - \mu)\, r^{n-1}(1-r). \tag{29}$$

So, if $E(Y(0)) = m_0$ is less than μ one will, more often than not, get an initial inhibition y less than μ and observe upward trend.

SPEARMAN'S UNIVERSAL FACTORS

The three universal factors postulated by Spearman (1927, page 327)—the general factor g, *perseveration*, and *oscillation*—can be identified as a natural consequence of the inhibition process postulated in the present model.

The general factor **g** (see Spearman, 1927, p. 75) is related to the level of performance in a test and, for example, not to the variability of the performance. The general level of the reaction times can be defined as the expectation of the reaction time when the process has become stationary, that is $E(T_n) = A(1 + a_1 a_0^{-1})$. Spearman explicitly states, when referring to **g**, that "...that which this magnitude measures has not been defined by declaring what it is like, but only by pointing out where it can be found." (Spearman, 1927, p. 75). The question of "what it is like" might now be tentatively answered in terms of the PI inhibition model as the product of A and $(a_0 + a_1)/a_0$, or, taking the logarithm of $E(T_n)$, as the sum of $\ln(A)$ and $\ln((a_0 + a_1)/a_0)$. This fits in nicely with Spearman: "After intelligence, the most widely supported

interpretation of **g** seems to be as the power of attention." (Spearman, 1927, p. 88). The concepts "intelligence" and "power of attention" can now be defined in terms of the two parameters of the inhibition process. "Intelligence" corresponds to the parameter A, which is inversely related to the speed of work. "Power of attention" corresponds to the ratio a_1/a_0. If **g** is defined as the product of A and $(a_0+a_1)/a_0$, one has to keep in mind that A itself is related to accuracy (number of correct responses). It is usually assumed that there exists a trade-off relationship between speed and accuracy. So, actually, one might consider **g** as a composition of three factors: A (or speed), accuracy, and power of attention (or mental effort), expressed in the ratio a_1/a_0.

It is generally assumed that these three factors are all interdependent; e.g., according to Thurstone (1937), speed and accuracy are dependent on effort (or motivation). Spending more effort actually implies spending more mental energy; that is, the rate of inhibition increase during work intervals will be smaller, which implies that the ratio a_1/a_0 decreases. Taken together, the interdependency between the three factors speed, accuracy, and effort might be formulated as follows:

(1) When effort (visible in the ratio a_1/a_0) remains constant, then there exists a trade-off relationship between speed and accuracy.

(2) When the time needed to accomplish the task remains constant, that is, speed remains constant, then any change in accuracy will be accompanied by a change in effort. The effect of a change in effort can be measured by the ratio a_1/a_0. When a subject wants to work more accurately without any loss in speed, then he/she has to spend more energy. As a result, during work intervals, the loss of energy at each moment of time will be at a lower level. This is equivalent to the assertion that the rate of inhibition increase during work intervals will be smaller.

(3) When accuracy remains constant, then any change in speed will be accompanied with a change in effort, that is in the ratio a_1/a_0. When a subject wants to work faster without any loss in accuracy, he/she has to spend more energy. As a result the rate of inhibition increase during work intervals will be smaller.

If these postulates are true[1] then it is possible to examine the relationship between speed, accuracy, and effort, with a view to appraising mental ability or **g** independent of speed, accuracy, and effort.

[1] A re-analysis of the data from two experiments (the Bourdon experiment and the Pauli experiment) described in van Breukelen et al. (1987) shows corroborating evidence. These results will be published shortly.

In discussing *oscillation* Spearman remarks that "the output of almost any kind of continuous work will throughout exhibit fluctuations that cannot be attributed to the nature of the work, but only to the worker himself." (Spearman, 1927, p. 321). The size of these fluctuations can be expressed in terms of the variance of the reaction times when the process is in its stationary state (formula 24)). Spearman's law of inertia, which he postulated to account for the phenomenon of *perseveration*, can be seen as follows: if the initial inhibition at the beginning of the task deviates from its stationary value, then it will take some time before the stationary state will be reached. The speed at which this happens depends on the trend factor e^{-bA}. So, it seems obvious to define perseveration as e^{-bA}.

The general factor **g** and the factors oscillation and perseveration are, according to the previously given definitions, all dependent on the latent parameters A, a_0, a_1, c_0, and c_1. However, these are the parameters which govern the underlying inhibition process and in that sense they are more fundamental than the observable statistics such as the stationary mean, the stationary variance, and the growth factor in the exponential trend curve. It therefore seems more appropriate in future ability measurement to use direct estimates for these parameters instead of the usual statistics such as the stationary mean.

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